THE QUAIDE MILLETH COLLEGE FOR MEN - SHIFT-II.

Question Bank.

Staff:- R. SURIYA BANU. - I BSC C/S.

R. BASKARAN. - I BCA

Class & subject:- I BSC C/S./ I BCA.

Allied mathematics - II.

(Common paper).

unit - I — Allied mathematics — II

II Semester — SBAMN.

Unit - 1.

2 Mark

1) Define Fourier Series.

If $f(x)$ is a periodic Function, then it can be represented by an infinite Series called Fourier Series as $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$; in the interval $c < x < 2\pi$ of period 2π, where a_0, a_n, and b_n are called Fourier Coefficients.

$$a_0 = \frac{1}{\pi} \int_c^{c+2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \cos nx\,dx$$

$$b_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \sin nx\,dx$$

2) Define Bernoulli Formula For integration by parts.

In case integration is to be used successfully we use Bernoulli's Formula

i.e. $\int u\,dv = uv = u'v_1 + u''v_2 - u'''v_3 + \ldots$

where $u', u'', u''' \ldots$ denotes successive differentiation of u and $v_1, v_2, v_3 \ldots$ successive integration of v.

3) Evaluate $\int x^2 e^{3x}\,dx$

choose $u = x^2$ and $dv = e^{3x}dx$; $v = \frac{e^{3x}}{3}$

$$\int x^2 e^{3x}\,dx = uv - u'v_1 + u''v_2 - u'''v_3 + \dots$$

$$= +x^2\frac{e^{3x}}{3} - 2x\frac{e^{3x}}{9} + 2\frac{e^{3x}}{27}$$

4) Evaluate $\int x^3 \cos 2x\,dx$

choose $u = x^3$ and $dv = \cos 2x\,dx$; $v = \frac{\sin 2x}{2}$

$$\int x^3 \cos 2x\,dx = uv - u'v_1 + u''v_2 - u'''v_3 + \dots\dots$$

$$= x^3\frac{\sin 2x}{2} - 3x^2\left(\frac{-\cos 2x}{4}\right) + 6x\left(\frac{-\sin 2x}{8}\right) - 6\left(\frac{\cos 2x}{16}\right)$$

$$= \frac{x^3}{2}\sin 2x + \frac{3}{4}x^2\cos 2x - \frac{3}{4}x\sin 2x - \frac{3}{8}\cos 2x$$

5) Evaluate $\int_0^{\pi/2} \sin^8 dx$

$$\int_0^{\pi/2} \sin^8 dx = \left(\frac{8-1}{8}\right)\left(\frac{8-3}{8-2}\right)\left(\frac{8-5}{8-4}\right)\left(\frac{8-7}{8-6}\right)\frac{\pi}{2}$$

$$= \left(\frac{7}{8}\right)\left(\frac{5}{6}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\frac{\pi}{2}$$

$$= \frac{35\pi}{256}$$

6) Evaluate $\int_0^{\pi/2} \cos^6 dx$

$$\int_0^{\pi/2} \cos^6 dx = \left(\frac{6-1}{6}\right)\left(\frac{6-3}{6-2}\right)\left(\frac{6-5}{6-4}\right)\frac{\pi}{2}$$

$$= \left(\frac{5}{6}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\frac{\pi}{2}$$

$$= \frac{5\pi}{32}$$

5 Mark

1. Expand $f(x) = x$ as sine series in the interval $(0, \pi)$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x \sin nx \, dx$$

$$b_n = \frac{2}{\pi}\left[(x)\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= -\frac{2}{\pi}\left[\left(\frac{\pi \cos n\pi - 0}{n}\right) - (0-0)\right]$$

$$= \frac{-2(-1)^n}{n}$$

Sine series $f(x) = \sum_{n=1}^{\infty} b_n \sin nx = \sum_{n=1}^{\infty} \frac{-2(-1)^n}{n} \sin nx$

$$f(x) = 2\left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \dots\right]$$

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2. Find a cosine series for the function $f(x) = \pi - x$ in the interval $(0, \pi)$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} (\pi - x)\, dx$$

$$= \frac{2}{\pi}\left(\frac{(\pi-x)^2}{-2}\right)_0^{\pi} = \frac{2}{\pi}\left(0 - \frac{\pi^2}{-2}\right) = \pi$$

$$a_0 = \pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx \, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} f(\pi - x) \cos nx \, dx$$

$$= \frac{2}{\pi}\left[(\pi - x)\left(\frac{\sin nx}{n}\right) - (-1)\left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[(0-0) - \left[\frac{(-1)^n - 1}{n^2}\right]\right]$$

Since $\sin n\pi = 0$ and $\cos n\pi = (-1)^n$, $\sin 0 = 0$ and $\cos 0 = 1$

$$a_n = -\frac{2}{\pi}\left[\frac{(-1)^n - 1}{n^2}\right]$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$= \frac{\pi}{2} - \sum_{n=1}^{\infty} \frac{2\left[(-1)^n - 1\right]}{n^2} \cos nx$$

$$= \frac{\pi}{2} + \frac{4}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} \cdots\right]$$

3) obtain a reduction formula for $\int x^n e^{ax} dx$

Let $I_n = \int e^{ax} x^n dx$

using integration by parts Formula.

$$u = x^n; \quad du = nx^{n-1} dx$$

$$dv = e^{ax} dx; \quad v = \frac{e^{ax}}{a}$$

$$I_n = uv - \int v du$$

$$= \frac{x^n e^{ax}}{a} - \int \frac{e^{ax}}{a} nx^{n-1} dx$$

$$= \frac{x^n e^{ax}}{a} - \frac{n}{a}\int e^{ax} x^{n-1} dx$$

$I_n = \frac{x^n e^{ax}}{a} - \frac{n}{a} I_{n-1}$ is the reduction Formula for $\int e^{ax} x^n dx$

4) obtain the reduction of $\int \sin^n x\,dx$

Let $I_n = \int \sin^n x\,dx$

$$I_n = \int \sin^{n-1} x \sin x\,dx$$

$$= uv - \int v\,du$$

$u = \sin^{n-1} x$; $dv = \sin x\,dx$; $v = -\cos x$

$$= \sin^{n-1} x(-\cos x) - \int (-\cos x)(n-1)\sin^{n-2} x \cos x\,dx$$

$$= -\sin^{n-1} x \cos x + (n-1)\int \cos^2 x \sin^{n-2} x\,dx$$

$$= -\sin^{n-1} x \cos x + (n-1)\int (1-\sin^2 x)\sin^{n-2} x\,dx$$

$$= -\sin^{n-1} x \cos x + (n-1)\int (\sin^{n-2} x - \sin^n x)\,dx$$

$$= -\sin^{n-1} x \cos x + (n-1)[I_{n-2} - I_n]$$

$$I_n = -\sin^{n-1} x \cos x + (n-1)[I_{n-2}] - (n-1)[I_n]$$

$$I_n + (n-1)I_n = -\sin^{n-1} x \cos x + (n-1)I_{n-2}$$

$$nI_n = -\sin^{n-1} x \cos x + (n-1)I_{n-2}$$

$$I_n = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} I_{n-2}$$

5) Evaluate $\int_0^{\pi/2} \sin^5 x \cos^6 x\, dx$ $m=5$ $n=6$

$$\int_0^{\pi/2} \sin^5 x \cos^6 x\, dx = \frac{(6-1)}{(m+6)} \frac{(6-3)}{(m+6-2)} \frac{(6-5)}{(m+6-4)} \cdots$$

$$= \frac{(5)}{(m+6)} \frac{(3)}{(m+4)} \frac{1}{(m+2)} \left[\frac{(m-1)}{m} \frac{(m-3)}{(m-2)} \cdots\right]$$

$$= \frac{(5)}{(5+6)} \frac{(3)}{(5+4)} \frac{1}{(5+2)} \left[\frac{(5-1)}{5} \frac{(5-3)}{(5-2)}\ 1\right]$$

$$= \frac{5}{11} \frac{3}{9} \frac{1}{7} \left[\frac{4}{5} \frac{2}{3}\ 1\right] = \frac{8}{693}$$

6) Evaluate the Following $\int_0^{\pi/2} \sin^6 x \cos^5 x\, dx$

$$\int_0^{\pi/2} \sin^6 x \cos^5 x\, dx = \frac{(5-1)}{(m+5)} \frac{(5-3)}{m+5-2} \cdots$$

$$= \frac{(4)}{(m+5)} \frac{(2)}{(m+3)} \frac{1}{(m+1)}$$

$$= \frac{(4)}{(6+5)} \frac{(2)}{(6+3)} \frac{1}{(6+1)}$$

$$= \frac{8}{693}$$

10 Mark

1) Find the Fourier series to the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval $(0, 2\pi)$

given $f(x) = \frac{1}{2}(\pi - x)$

we know that Fourier Series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where,

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \frac{1}{2}(\pi - x)\,dx$$

$$= \frac{1}{\pi}\left[\frac{(\pi - x)^2}{(-1)(2)}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{(-\pi)^2 - \pi^2}{-2}\right]$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \frac{1}{2}(\pi - x)\cos nx\,dx$$

Using Bernoulli's formula for integration of parts.

$$a_n = \frac{1}{2\pi}\left[(\pi - x)\left(\frac{\sin nx}{n}\right) - (-1)\left(\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[(\pi - 2\pi)\frac{\sin 2n\pi}{n} - (\pi - 0)\frac{\sin 0}{n} - \left(\frac{\cos 2n\pi}{n^2} - \frac{\cos 0}{n^2}\right)\right]$$

$$= \frac{1}{2\pi}\left[(-\pi)0-(\pi)0-\left(\frac{1}{n^2}-\frac{1}{n^2}\right)\right]$$

$$=0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx \, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \frac{1}{2}(\pi - x)\sin nx \, dx$$

Using Bernoulli's Formula for integration of parts.

$$b_n = \frac{1}{2\pi}\left[(\pi-x)\left(-\frac{\cos nx}{n}\right)-(-1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[(\pi-2\pi)\left(-\frac{\cos 2n\pi}{n}\right)-(\pi-0)\left(-\frac{\cos 0}{n}\right)\right.$$

$$\left.-\left(\frac{\sin 2n\pi}{n^2}-\frac{\sin 0}{n^2}\right)\right]$$

$$= \frac{1}{2\pi}\left[(-\pi)\left(-\frac{1}{n}\right)-(\pi)\left(-\frac{1}{n}\right)-\left(\frac{0}{n^2}-\frac{0}{n^2}\right)\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{2\pi}{n}\right)\right] = \frac{1}{n}$$

Therefore

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$f(x) = 0 + 0 + \sum_{n=1}^{\infty} \frac{1}{n}\sin nx$$

$$f(x) = \frac{1}{2}(\cancel{2}\pi - x) = \frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \frac{\sin 4x}{4} + \cdots$$

2) prove that $f(x)=\frac{\pi^3}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n\cos nx}{n^2}-2\sum_{n=1}^{\infty}\frac{(-1)^n\sin nx}{n}$ is Fourier Series to the function $f(x)=x+x^2$ in the interval $(-\pi,\pi)$ Deduce $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}\cdots$

Given $f(x)=x+x^2$

we know that fourier series is given by

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx$$

$$=\frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)dx$$

$$=\frac{1}{\pi}\int_{-\pi}^{\pi}x\,dx+\frac{1}{\pi}\int_{-\pi}^{\pi}x^2dx$$

$$=0+\frac{2\pi}{\pi}\int_{0}^{\pi}x^2dx$$

$$=0+\frac{2}{\pi}\left(\frac{x^3}{3}\right)_0^{\pi}$$

$$=\frac{2}{\pi}\left(\frac{x^3}{3}-\frac{0}{3}\right)$$

$$=\frac{2\pi^2}{3}$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx$$

$$=\frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\cos nx\,dx$$

$$=\frac{1}{\pi}\int_{-\pi}^{\pi}x\cos nx\,dx+\frac{1}{\pi}\int_{-\pi}^{\pi}x^2\cos nx\,dx$$

Let $g(x) = x \cos nx$

$$g(-x) = -x \cos(-nx) = -g(x)$$

Therefore, the integrand in I integral is odd and its Value is Zero. In the Second integral in $g(-x) = (-x)^2 \cos(-nx) = x^2 \cos nx = g(x)$ and the II integral is even. (refer result (v))

$$a_n = \frac{2}{\pi} \int_0^{\pi} x^2 \cos nx \, dx$$

$$= \frac{2}{\pi} \left[(x^2)\left(\frac{\sin nx}{n}\right) - (2x)\left(-\frac{\cos nx}{n^2}\right) + (2)\left(\frac{-\sin nx}{n^3}\right) \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[(0-0) + \left(\frac{2\pi \cos n\pi}{n^2} - 0\right) - (0-0) \right]$$

Since $\sin n\pi = 0$ and $\cos n\pi = (-1)^n$; $\sin 0 = 0$ and $\cos 0 = 1$

$$a_n = 4\left[\frac{(-1)^n}{n^2}\right]$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2) \sin nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin nx \, dx + \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \sin nx \, dx$$

Let $g(x) = x \sin nx$

$$g(-x) = -x \sin(-nx) = g(x)$$

Therefore, the integrand in I integral is even.

In the second Integral $g(-x) = (-x)^2 \sin(-nx) = -x^2 \sin nx$ $= -g(x)$ and the II integral is odd and its value is zero.

$$b_n = \frac{2}{\pi}\int_0^{\pi} x \sin nx \, dx$$

$$= \frac{2}{\pi}\left[(x)\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{-2}{\pi}\left[\left(\frac{\pi \cos n\pi}{n} - 0\right) - (0-0)\right]$$

Since. $\sin n\pi = 0$ and $\cos n\pi = (-1)^n$, $\sin 0 = 0$ and $\cos 0 = 1$

$$b_n = -2\left[\frac{(-1)^n}{n}\right]$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$x + x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n \cos nx}{n^2} + \sum_{n=1}^{\infty} \frac{-2(-1)^n \sin nx}{n}$$

$$= \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2} - 2\sum_{n=1}^{\infty} \frac{(-1)^n \sin nx}{n}$$

$$= \frac{\pi^2}{3} + \left[-\frac{\cos x}{1^2} + \frac{\cos 2x}{2^2} - \frac{\cos 3x}{3^2} + \dots\right]$$

$$= -2\left[-\frac{\sin x}{1} + \frac{\sin 2x}{2} - \frac{\sin 3x}{3} + \dots\right] \quad (1)$$

put $x = \pi$ in (1)

$$f(x) = \frac{\pi^2}{3} + 4\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\right] \quad (2)$$

The value of the function at the end points is the average of $f(-\pi)$ and $f(\pi)$

$$f(x) = \frac{-\pi+\pi^2+\pi+\pi^2}{2}$$

$$= \pi^2$$

$$\therefore \pi^2 = \frac{\pi^2}{3} + 4\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\right]$$

$$\pi^2 - \frac{\pi^2}{3} = 4\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\right]$$

Hence $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots$

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3 Evaluate $\int_0^{\pi/2} \sin^n dx$, when n is even and odd

$$I_n = \int_0^{\pi/2} \sin^n x \, dx$$

From example 5, $I_n = -\left(\frac{\sin^{n-1} x \cos x}{n}\right)_0^{\pi/2} + \frac{n-1}{n} I_{n-2}$

$$I_n = -[0-0] + \left(\frac{n-1}{n}\right) I_{n-2}$$

$I_n = \left(\frac{n-1}{n}\right) I_{n-2}$. This reduction formula enables us evaluate the integral for any positive n even or odd by replacing n by $n-2$ we get recursively the next integral

$$I_{n-2} = \left(\frac{n-2-1}{n-2}\right) I_{n-2-2}$$

$$I_{n-2} = \left(\frac{n-3}{n-2}\right) I_{n-4}$$

to Find the next integral replace n by n-4

$$I_{n-4} = \left(\frac{n-4-1}{n-4}\right) I_{n-4-2}$$

$$I_{n-4} = \left(\frac{n-5}{n-4}\right) I_{n-6} \qquad (3)$$

using (2) and (3) in (1)

$$I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \ldots I_0 \text{ or } I_1 \text{ accordingly}$$

as n is even or odd.

Case (i) when n is even, last integral is I_0

where $I_0 = \int_0^{\pi/2} dx = [x]_0^{\pi/2} = \pi/2$

Therefore, $\int_0^{\pi/2} \sin^n dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \cdots \frac{1}{2}\frac{\pi}{2}$

Case (ii) when n is odd last integral is I_1

where $I_1 = \int_0^{\pi/2} \sin x \, dx = [-\cos x]_0^{\pi/2}$

$$= -\left[\cos \pi/2 - \cos 0\right]$$

$$= 1$$

Therefore, $\int_0^{\pi/2} \sin^n dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \cdots \frac{2}{3}(1)$

Note $\int_0^{\pi/2} \cos^n dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \cdots \frac{1}{2}\frac{\pi}{2}$ if n is even

$\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \cdots \frac{2}{3} 1$ if n is odd Left as exercise to studen

UNIT - 2

2 marks

① difine clairauts form?

this equation $z = px + qy + f(p,q)$ is called clairauts form it's complete solution is $z = ax + by + f(a,b)$

② solve $3y^2 p = q$

the differential $F(y, p, q) = 0$

here x is absent then take $p = a$

$$3y^2 p = q$$

$$3y^2 a = q$$

$$q = 3y^2 a$$

$dz = p\,dx + q\,dy$

$dz = a\,dz + 3y^2 a\,dy$

$$\boxed{z = ax + \frac{3y^3}{9} a + c}$$

③ Solve $p = q^2$

$z = ax + by + c$; $\frac{dz}{dx} = a$, $\frac{dz}{dy} = b$

$p = a$, $q = b$ $\quad a = b^2$; $b = \sqrt{a}$

$$\boxed{z = ax + \sqrt{a}\,y + c}$$

4) Find the auxillary equation to $(D^2+a^2)y = \sin ax$.

the auxillary equation

$$m^2+a^2=0$$

$$m^2 = -a^2$$

$$m^2 = i^2 a^2$$

$$\boxed{m = \pm ia}$$

$$C.F = e^{\alpha x}[A\cos ax + B\sin ax]$$

$$\boxed{C.F = A\cos ax + B\sin ax}$$

5) defind the P.I ?

to find the Particular Integral

for $f(x) = e^{ax}$

$$P.I = \frac{1}{f(D)} e^{ax}$$

6) solve $P = \tan(y-Px)$

$$P = \tan(y-Px)$$

$$\tan^{-1}P = y - Px$$

$$y = Px + \tan^{-1}P$$

the form of this equation is clairaut's equation $y = Px + f(P)$

the solution is $y = cx + f(c)$

therefore the general solution is $y = cx + \tan^{-1}c$

5 marks

1) solve $2\frac{d^2y}{dx^2} - \frac{dy}{dx} - y = 4e^x$

the auxillary equation of the given equation is

$2D^2y - Dy - y = 4e^x$

$(2D^2 - D - 1)\,y = 4e^x$

$2m^2 - m - 1 = 0$

$2m^2 - 2m + m - 1 = 0$

$2m(m-1) + 1(m-1) = 0$

$2m + 1 = 0$ | $m - 1 = 0$

$2m = -1$ | $\boxed{m_2 = 1}$

$\boxed{m_1 = -1/2}$

$C.F = Ae^{-1/2\,x} + Be^x$

To find P.I :

$P.I = \frac{1}{2D^2 - D - 1} 4e^x$ $\quad \boxed{D = 1}$

$= \frac{1}{2-1-1} 4e^x$

Dinaminater 0 using x

$= \frac{1}{2D^2 - D - 1} 4e^x$ $\quad$ D=1

$= x \frac{1}{4D-1}$ $\quad$ D = 1

$$= \frac{x4e^x}{3}$$

therefore the complete solution is

$$\boxed{y = Ae^{-1/2 x} + Be^x + \frac{x4e^x}{3}}$$

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2) solve $(D^2 - 3D + 2)y = \sin 2x$

the auxillary equation is the equation of

$$m^2 - 3m + 2 = 0$$

$$m^2 - m - 2m + 2 = 0$$

$$m(m-1) - 2(m-1) = 0$$

$m - 2 = 0$ | $m - 1 = 0$

$\boxed{m = 2}$ | $\boxed{m = 1}$

$$C.F = Ae^{2x} + Be^x$$

To find P.I

$$P.I = \frac{1}{f(D)} \sin 2x$$

$$= \frac{1}{D^2 - 3D + 2} \sin 2x$$

$$D^2 = -4$$

$$= \frac{1}{-4 - 3D + 2} \sin 2x$$

$$= \frac{\sin 2x}{-3D - 2}$$

multiplying by $-3D+2$ on both Nr & Dr

$$= \frac{-3D+2}{(-3D-2)(-3D+2)} \sin 2x$$

$$= \frac{-6\cos 2x + 2\sin 2x}{9D^2-4} \qquad \boxed{D^2=-4}$$

$$= \frac{-6\cos 2x + 2\sin 2x}{-40}$$

$$P.I = \frac{6\cos 2x - 2\sin 2x}{40}$$

therefore the complete solution : $y = C.F + P.I$

$$\boxed{y = Ae^{2x} + Be^{x} + \frac{6\cos 2x - 2\sin 2x}{40}}$$

3) solve $z = pq$

the differential form of $F(z, p, q) = 0$

here x, y is absent then take p and q

Constant,

Let: $q = ap$, $z = ap^2$

using in the total differential:

$$dz = p\,dx + q\,dy$$

$$dz = \frac{\sqrt{z}}{\sqrt{a}}\,dx + \sqrt{a}\,.\sqrt{z}\,dy$$

$$dz = \sqrt{z}\left[\frac{dx}{\sqrt{a}} + \sqrt{a}\,dy\right]$$

$$\frac{dz}{\sqrt{z}} = \frac{dx}{\sqrt{a}} + \sqrt{a}\,dy$$

integrating :

$$\int \frac{dz}{\sqrt{z}} = \int \frac{dx}{\sqrt{a}} + \int \sqrt{a}\,dy$$

$$\boxed{2\sqrt{z} = \frac{x}{\sqrt{a}} + \sqrt{a}\,y + C}$$

(4) $p^2 + q^2 = x^2 + y^2$ to solve the equation.

$$p^2 + q^2 = x^2 + y^2$$

$$p^2 - x^2 = y^2 - q^2 = a^2$$

$p^2 - x^2 = a^2$	$y^2 - q^2 = a^2$
$p^2 = a^2 + x^2$	$q^2 = y^2 - a^2$
$p = \sqrt{a^2 + x^2}$	$q = \sqrt{y^2 - a^2}$

$$dz = p\,dx + q\,dy.$$

$$dz = \sqrt{a^2+x^2}\, dx + \sqrt{y^2-a^2}\, dy$$

Integration:

$$\int dz = \int \sqrt{a^2+x^2}\, dx + \int \sqrt{y^2-a^2}\, dy$$

$$z = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)$$

$$+ \frac{y}{2}\sqrt{y^2-a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{y}{a}\right) + c$$

---

5) solve $xp + yq = z$

by the diffenition of Lagranges.

$$Pp + Qq = R$$

$$P = x \;;\; Q = y \;;\; R = z$$

then auxillary equation of the equation in

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

choosing any two ratio

$$\frac{dx}{x} = \frac{dy}{y}$$

taking integration

$$\int \frac{dx}{x} = \int \frac{dy}{y}$$

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$$\log x = \log y + \log c_1$$

$$\log x - \log y = \log c_1$$

$$\log x/y = \log c_1$$

$$\boxed{x/c = c_1}$$

other two ratio:

$$\int \frac{dy}{y} = \int \frac{dz}{z}$$

$$\log y = \log z + \log c_2$$

$$\log y - \log z = \log c_2$$

$$\log y/z = \log c_2$$

$$\boxed{y/z = c_2}$$

therefore the complete solution is:

$$c_1 = u \quad ; \quad c_2 = v$$

$$F(u, v) = 0$$

$$\boxed{F(x/y,\ y/z) = 0}$$

10 marks

1) solve $(D^2-7D+6)y = e^{2x}\sin 3x$

the auxillary equation is the equation of

$$m^2-7m+6=0$$

$$m^2-m-6m+6=0$$

$$m(m-1)-6(m-1)=0$$

$m-1=0$ | $m-6=0$

$m_1=1$ | $m_2=6.$

$$C.F = Ae^{m_1x}+Be^{m_2x}$$

$$\boxed{C.F = Ae^{6x}+Be^{x}}$$

To find P.I

$$P.I = \frac{1}{f(D)}e^{2x}\sin 3x$$

$$= \frac{1}{D^2-7D+6}e^{2x}\sin 3x$$

$D \Rightarrow (D+2)$

$$= e^{2x}\frac{1}{(D+2)^2-7(D+2)+6}\sin 3x$$

$$= e^{2x}\frac{1}{D^2+4+4D-7D+14+6}\sin 3x$$

$$= e^{2x} \frac{1}{D^2 - 3D - 4} \sin 3x$$

$$\boxed{D^2 = -9}$$

$$= e^{2x} \frac{1}{-9 - 3D - 4} \sin 3x$$

$$= e^{2x} \frac{1}{-13 - 3D} \sin 3x$$

$$= e^{2x} \frac{(-13 + 3D)}{(-13 - 3D)(-13 + 3D)} \sin 3x$$

$$= e^{2x} \frac{-13 \sin 3x + 9 \cos 3x}{169 - 9(D^2)}$$

$$= e^{2x} \frac{(-13 \sin 3x + 9 \cos 3x)}{169 - 9(-9)}$$

$$P.I = e^{2x} \frac{(-13 \sin 3x + 9 \cos 3x)}{250}$$

$$y = C.F + P.I$$

$$y = Ae^{6x} + Be^{x} + e^{2x} \frac{(-13 \sin 3x + 9 \cos 3x)}{250}$$

2) solve $(mz - ny)p + (nx - lz)q = (ly - mx)$

By the diffenition of the lagranges

$$Pp + Qq = R$$

Here

$$P = mz - ny \ ; \ Q = nx - lz \ ; \ R = ly - mx$$

the auxillary equation of the equation is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$$

using multipliers (l, m, n) :

$$\frac{l\,dx}{lmz - lny} = \frac{m\,dy}{mnx - mlz} = \frac{n\,dz}{nly - nmx}$$

$$\frac{l\,dx + m\,dy + n\,dz}{\cancel{lmz} - \cancel{lny} + \cancel{mnx} - \cancel{mlz} + \cancel{nly} - \cancel{nmx}}$$

$$l\,dx + m\,dy + n\,dz = 0$$

taking Integration : $\int l\,dx + \int m\,dy + \int n\,dz = 0$

$$\boxed{lx + my + nz = c_1}$$

Using multipliers (x, y, z)

$$\frac{x\,dx}{xmz - xny} = \frac{y\,dy}{ynx - ylz} = \frac{z\,dz}{zly - zmx}$$

$$\frac{x\,dx + y\,dy + z\,dz}{\cancel{xmz} - \cancel{xny} + \cancel{ynx} - \cancel{ylz} + \cancel{zly} - \cancel{zmx}}$$

$$x\,dx + y\,dy + z\,dz = 0$$

takeing integration:

$$\int x\,dx + \int y\,dy + \int z\,dz = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{c_2}{2}$$

$$\boxed{x^2 + y^2 + z^2 = c_2}$$

the complete solution:

$$c_1 = u \;;\; c_2 = v$$

$$F(u, v) = 0$$

$$\boxed{F\left(lx + my + nz,\; x^2 + y^2 + z^2\right) = 0}$$

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3) Solve $\frac{dy}{dx} - \frac{dx}{dy} = x/y - y/x$

Put $\frac{dy}{dx} = p$ then the equation takes the form:

$$p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$$

$$\therefore \frac{p^2-1}{p} = \frac{x^2-y^2}{xy}$$

$$xyp^2 - (x^2-y^2)p - xy = 0$$

the equation is solvable for p

$$p = \frac{(x^2-y^2) \pm \sqrt{(x^2-y^2)^2 + 4x^2y^2}}{2xy}$$

$$= \frac{(x^2-y^2) \pm (x^2+y^2)}{2xy}$$

$$p = \frac{2x^2}{2xy} \; ; \; p = \frac{-2y^2}{2xy}$$

$$p = x/y \; ; \; p = -y/x$$

$$\frac{dy}{dx} = x/y \; ; \; \frac{dy}{dx} = \frac{-y}{x}$$

$y\,dy = x\,dx$; $\dfrac{dy}{y} = -\dfrac{dx}{x}$

Integrating,

$\dfrac{y^2}{2} = \dfrac{x^2}{2} + c/2$; $\log y = -\log y + \log c$

$y^2 - x^2 - c = 0$; $xy - c = 0$

$\therefore\ (y^2 - x^2 - c)(xy - c) = 0$

UNIT - 3

2 marks

① Define the laPlace Transforms ?

the laplace transforms is

$$\int_0^\infty e^{-st} f(t)\,dt = L[f(t)] = F(s)$$

② Define the Linearty praperty ?

the Linearty praperty

$$L[a f(t) + b g(t)] = aL[f(t)] + bL[g(t)]$$

③ Evaluate $\int_0^\infty e^{-3t} \sin 4t \, dt$

By the deffinision of the Laplace ~~tran~~ transform

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

$$L[\sin 4t] = \frac{4}{s^2+16} \quad s = 3$$

$$L[e^{-3t} \sin 4t] = \frac{4}{9+16}$$

$$\boxed{\int_0^\infty e^{-3t} \sin 4t \, dt = 4/25}$$

④ Define shift theorems :

(i) $L[e^{at} f(t)] = F[s-a]$

(ii) $L[e^{-at} f(t)] = F[s+a]$

(5) Define $L^{-1} \dfrac{s^2 - a^2}{(s^2+a^2)^2}$

$$L^{-1} \frac{s^2 - a^2}{(s^2+a^2)^2} = t \cos at$$

(6) Evaluate $L[e^t \sin t]$

$$L[e^t \sin t]$$

$$L[\sin t] = \frac{1}{s^2+1}$$

$$L[e^t \sin t] = \frac{1}{s^2+1} \quad s \Rightarrow s-1$$

$$L[e^t \sin t] = \frac{1}{(s-1)^2+1}$$

$$= \frac{1}{s^2+1-2s+1}$$

$$\boxed{L[e^t \sin t] = \frac{1}{s^2-2s+2}}$$

(7) Define $L[y'']$

$$L[y''] = s^2 L[y] - s\,y(0) - y'(0)$$

5 marks

① Prove $L[\sin at] = \dfrac{a}{(s^2+a^2)}$

Using $L(f(t)) = \int_0^{\infty} e^{-st} f(t)\, dt$

$$L[\sin at] = \int_0^{\infty} e^{-st} \sin at\, dt$$

using the Euler formula $e^{ix} = \cos x + i\sin x$

$$L[\sin at] = \text{Imaginary Part} \int_0^{\infty} e^{-st} e^{iat}\, dt$$

$$= \text{I.P.} \int_0^{\infty} e^{-(s-ia)t}\, dt$$

$$= \text{I.P} \frac{1}{(s-ia)}$$

$$= \text{I.P} \frac{s+ia}{(s^2+a^2)}.$$

$$L[\sin at] = \frac{a}{(s^2+a^2)}$$

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(2) Evaluate $L[e^t(\sin at + \cos t)]$

$$L[e^t(\sin t + \cos t)] \Rightarrow L[e^t \sin t] + L[e^t \cos t]$$

To find: $L[e^t \sin t]$

$$L[\sin t] = \frac{1}{s^2+1}$$

$$L[e^t \sin t] = \frac{1}{s^2+1} \quad s \to s-1$$

$$= \frac{1}{(s-1)^2+1}$$

$$= \frac{1}{s^2+1-2s+1}$$

$$\boxed{L[e^t \sin t] = \frac{1}{s^2-2s+2}}$$

To find: $L[e^t \cos t]$

$$L[\cos t] = \frac{s}{s^2+1}$$

$$L[e^t \cos t] = \frac{s}{s^2+1} \quad s \to s-1$$

$$\Rightarrow \frac{s-1}{(s-1)^2+1} \Rightarrow \frac{s-1}{s^2+1-2s+1}$$

$$\boxed{L[e^t \cos t] = \frac{s-1}{s^2-2s+2}}$$

$$L[e^t \sin t] + L[e^t \cos t] = \frac{1}{s^2-2s+2} + \frac{s-1}{s^2-2s+2}$$

$$L[e^t(\sin t \cos t)] = \frac{1+s-1}{s^2-2s+2}$$

$$L\left[e^{t}(\sin t+\cos t)\right] = \frac{s}{s^2-2s+2}$$

③ Evaluate $L\left[te^{-2t}\cos 3t\right]$

To Find:

$$L[\cos 3t] = \frac{s}{s^2+9}$$

$$L\left[e^{-2t}\cos 3t\right] = \left[\frac{s}{s^2+9}\right]_{s\to s+2}$$

$$L\left[e^{-2t}\cos 3t\right] = \frac{s+2}{(s+2)^2+9}$$

$$= \frac{s+2}{s^2+4+4s+9}$$

$$L\left[e^{-2t}\cos 3t\right] = \frac{s+2}{s^2+4s+13}$$

$$L\left[te^{-2t}\cos 3t\right] = \frac{-d}{ds}F(s)$$

$$= \frac{-d}{ds}L\left[e^{-at}\cos 3t\right]$$

$$= \frac{-d}{ds}\left[\frac{s+2}{s^2+4s+13}\right]$$

$$= \left[\frac{s^2+4s+13(1)-(s+2)(2s+4)}{(s^2+4s+13)^2}\right]$$

$$= -\left[\frac{s^2+4s+13-2s^2-4s-4s+8}{(s^2+4s+13)^2}\right]$$

$$= -\left[\frac{-s^2-4s+5}{(s^2+4s+13)^2}\right]$$

$$\boxed{L\left[te^{-2t}\cos 3t\right] = \left[\frac{s^2+4s-5}{(s^2+4s+13)^2}\right]}$$

---

(4) Evaluate $L\left[\frac{e^{-at}-e^{-bt}}{t}\right]$

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s)\,ds$$

$$F(s) = L\left[e^{-at}-e^{-bt}\right]$$

$$= L\left[e^{-at}\right] - L\left[e^{-bt}\right]$$

$$F(s) = \frac{1}{s+a} - \frac{1}{s+b}$$

$$F(s) = \int_s^{\infty}\left(\frac{1}{s+a} - \frac{1}{s+b}\right)ds$$

$$\Rightarrow \left[\log(s+a) - \log(s+b)\right]_s^{\infty}$$

$$\Rightarrow \left[\log\frac{s+a}{s+b}\right]_s^{\infty}$$

$$\Rightarrow \log\left[\frac{s(1+a/\infty)}{s(1+b/\infty)}\right] - \log\frac{(s+a)}{(s+b)}$$

$$\Rightarrow \log(1) - \log\frac{s+a}{s+b}$$

$$\Rightarrow \log\frac{1}{\frac{s+a}{s+b}}$$

$$\Rightarrow \log\frac{s+b}{s+a}$$

$$\boxed{L\left[\frac{e^{-at}-e^{-bt}}{t}\right] = \log\frac{s+b}{s+a}}$$

---

(5) Find $L^{-1}\left[\frac{1}{s^2+5s+6}\right]$

$$L^{-1}\left[\frac{1}{s^2+5s+6}\right] \Rightarrow$$

$$\Rightarrow s^2+5s+6$$

$$\Rightarrow s^2+5s+6=0$$

$$\Rightarrow s^2+2s+3s+6=0$$

$$\Rightarrow s(s+2)\,3(s+2)=0$$

$$= (s+3)(s+2)$$

$$L^{-1}\left[\frac{1}{(s+3)(s+2)}\right]$$

Resolve into portial fraction form

$$\frac{1}{(s+3)(s+2)} = \frac{A}{s+3} + \frac{B}{s+2}$$

multiplay on $(s+3)(s+2)$ both said :

$$1 = A(s+2) + B(s+3)$$

Put $s=-2$;

$$1 = 0 + B(-2+3)$$

$$\boxed{1 = B}$$

$s = -3$

$$1 = A(-3+2) + 0$$

$$1 = -A$$

$$\boxed{A = -1}$$

$$\frac{1}{(s+3)(s+2)} = \frac{-1}{s+3} + \frac{1}{s+2}$$

$$L^{-1}\left[\frac{1}{(s+3)(s+2)}\right] = -L^{-1}\left[\frac{-1}{s+3}\right] + L^{-1}\left[\frac{1}{s+2}\right]$$

$$\boxed{L^{-1}\left[\frac{1}{(s+3)(s+2)}\right] = -e^{-3t} + e^{-2t}}$$

(6) Prove that $L[t\, f(t)] = -\frac{d}{ds} F(s)$ where $F(s) = L[f(t)]$

$$L[f(t)] = -\frac{d}{ds} F(s)$$

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt = F(s)$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^\infty \frac{\partial}{\partial s} (e^{-st}) f(t)\, dt$$

$$= \int_0^\infty (-t e^{-st}) f(t)\, dt$$

$$= -\int_0^\infty (e^{-st})\, t\, f(t)\, dt$$

$$= -L[t\, f(t)]$$

$$\boxed{L[t\, f(t)] = -\frac{d}{ds} F(s)}$$

solve using laplace the following differential equation

$\frac{d^2y}{dt^2} + 25y = 10\cos 5t$ given $y(0) = 2$ $y'(0) = 0$

$$y'' + 25y = 10\cos 5t \quad y(0) = 2, \; y'(0) = 0$$

taking Laplace on both said :

$$L(y'') + 25L(y) = 10L\cos 5t$$

$$s^2L(y) - sy(0) - y'(0) + 25L(y) = 10\frac{s}{s^2+25}$$

$$s^2L(y) - 2s + 25L(y) = 10\frac{s}{s^2+25}$$

$$L(y)\left[s^2+25\right] = 10\frac{s}{s^2+25} + 2s$$

$$L(y) = \frac{10s}{(s^2+25)^2} + \frac{2s}{s^2+25}$$

$$y = L^{-1}\left[\frac{10s}{(s^2+25)^2}\right] + L^{-1}\left[\frac{2s}{s^2+25}\right]$$

$$y = L^{-1}\left[\frac{2(5s)}{(s^2+25)^2}\right] + 2L^{-1}\left[\frac{s}{s^2+25}\right]$$

$$\boxed{y = t\sin 5t + 2\cos 5t}$$

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10 marks

① Evaluate $L^{-1}\left[\dfrac{s+4}{s(s-1)(s^2+4)}\right]$

$$L^{-1}\left[\frac{s+4}{s(s-1)(s^2+4)}\right]$$

Resolve into partial fraction

$$\frac{s+4}{s(s-1)(s^2+4)} = \frac{A}{s} + \frac{B}{s-1} + \frac{Cs+D}{s^2+4}$$

$$s+4 = A(s-1)(s^2+4) + B(s)(s^2+4) + Cs(s)(s-1)$$
$$+ D(s)(s-1)$$

$s = 0$:

$$4 = A(-1)(4) + 0 + 0 + 0$$

$$\boxed{A = -1}$$

$s = 1$;

$$5 = 0 + B(1)(1+4) + 0 + 0$$

$$\boxed{B = 1}$$

comparing the co-efficient of s^3 :

$$0 = A + B + C$$

$$= \cancel{-1} \cancel{+1} + C$$

$$\boxed{C = 0}$$

comparing the co-efficient of s:

$1 = 4A + 4B - D$

$1 = \cancel{-4}\ A4 - D$

$\boxed{D = -1}$

$$\frac{s+4}{s(s-1)(s^2+4)} = \frac{-1}{s} + \frac{1}{s-1} - \frac{1}{s^2+4}$$

$$L^{-1}\left[\frac{s+4}{s(s-1)(s^2+4)}\right] = -L^{-1}\left[\frac{1}{s} + \frac{1}{s-1} - \frac{1}{s^2+4}\right]$$

$$= -L^{-1}\left[\frac{1}{s}\right] + L^{-1}\left[\frac{1}{s-1}\right] - L^{-1}\left[\frac{1}{s^2+4}\right]$$

$$= -1 + e^t - \frac{\sin 2t}{2}$$

$$\boxed{L^{-1}\left[\frac{s+4}{s(s-1)(s^2+4)}\right] = -1 + e^t - \frac{1}{2}\sin 2t}$$

(2) Solve $y'' - y = x^2 + x$ using Laplace transform given that $y(0) = y'(0) = 0$

Given that $y'' - y = x^2 + x$ $\quad y(0) = y'(0) = 0$

takeing laplace on both said

$$L[y'' - y] = L[x^2 + x]$$

$$L[y''] - L[y] = L[x^2] + L[x]$$

$$s^2 L[y] - s\,y(0) - y(0) - L(y) = \frac{2}{s^3} + \frac{1}{s^2}$$

$$s^2 L[y] - 0 - 0 - L[y] = \frac{2+s}{s^3}$$

$$L[y]\,[s^2 - 1] = \frac{2+s}{s^3}$$

$$L[y] = \frac{2+s}{s^3(s^2-1)}$$

$$y = L^{-1}\left[\frac{2+s}{s^3(s^2-1)}\right]$$

Resolve into Partial fraction:

$$\frac{s+2}{s^3(s-1)(s+1)} = A/s + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-1} + \frac{E}{s+1}$$

$$s+2 = A(s^2)\cancel{(s^3)}(s-1)(s+1) + B(s)(s-1)(s+1) + C(s-1)(s+1) + D(s^3)(s+1) + E(s^3)(s-1)$$

$\underline{\underline{s = 1}}$

$3 = A(1)(0)(2) + B(1)(0) + 0 + D(1)(2) + E(0)$

$3 = 2D$

$\boxed{D = 3/2}$

$s = -1$

$1 = A(0) + B(0) + C(0) + D(0) + E(-1)(-2)$

$1 = 2E$

$\boxed{E = 1/2}$

$s = 0$

$2 = C(-1)(1)$

$\boxed{C = -2}$

Comparing co-efficient of s^3

$0 = B + D - E$

$= B + 3/2 - 1/2$

$= B + 2/2$

$= B + 1$

$\boxed{B = -1}$

Comparing co-efficient of s^4

$0 = A + D + E$

$= A + 3/2 + 1/2 \Rightarrow A + \cancel{4}/\cancel{2} \Rightarrow A + 2$

$\boxed{A = -2}$

$$\frac{s+2}{s^3(s-1)(s+1)} = -\frac{2}{s} - \frac{1}{s^2} - \frac{2}{s^3} + \frac{3}{2}\left(\frac{1}{(s-1)}\right) + \frac{1}{2}\left(\frac{1}{(s+1)}\right).$$

$$L^{-1}\left[\frac{s+2}{s^3(s-1)(s+1)}\right] = -2L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s^2}\right] - 2L^{-1}\left[\frac{1}{s^3}\right] + \frac{3}{2}L^{-1}\left[\frac{1}{(s-1)}\right] + \frac{1}{2}L^{-1}\left[\frac{1}{s+1}\right]$$

$$= -2 - t - \frac{\cancel{2}t^2}{\cancel{2}} + \frac{3}{2}e^t + \frac{1}{2}e^{-t}$$

$$\therefore \boxed{y = -2 - t - t^2 + \frac{3}{2}e^t + \frac{1}{2}e^{-t}}$$

---

(3) solve $\frac{dx}{dt} + 2x - 3y = 0$; $\frac{dy}{dt} - 3x + 2y = 0$ given $x(0) = 0$, $y(0) = 2$

$x' + 2x - 3y = 0$

taking laplace on both said:

$L[x'] + 2L[x] - 3L[y] = 0$

$sL[x] - x(0) + 2L(x) - 3L(y) = 0$

$\boxed{L[x](s+2) - 3L(y) = 0}$ ---- (1)

$y' - 3x + 2y$

taking laplace on both said ;

$L[y'] - 3L(x) + 2L(y) = 0$

$sL(y) - y(0) - 3L(x) + 2L(y) = 0$

$sL(y) - 2 - 3L(x) + 2L(y) = 0$

$\boxed{L(y)(s+2) - 3L(x) = 2}$ ---- (2)

(1) $\times 3 \Rightarrow 3L(x)(s+2) - 9L(y) = 0$ ---- (3)

(2) $\times (s+2) \Rightarrow -3L(x)(s+2) + L(y)(s+2)^2 = 2(s+2)$ ---- (4)

$-9L(y) + L(y)(s+2)^2 = 2(s+2)$

$L(y)\left[(s+2)^2 - 9\right] = 2(s+2)$

$$L(y) = \frac{2(s+2)}{(s+2)^2 - 9}$$

$$y = L^{-1}\left[\frac{2(s+2)}{(s+2)^2 - 9}\right]$$

$$= 2L^{-1}\left[\frac{s+2}{(s+2)^2 - 9}\right]$$

$$= 2e^{-2t} L^{-1}\left[\frac{s}{s^2 - 9}\right]$$

$$\boxed{y = 2e^{-2t}\cosh 3t}$$

using the value of y in the differential we can get the value of x:

$$\frac{dy}{dt} - 3x + 2y = 0$$

$$3x = \frac{dy}{dt} + 2y$$

$$3x = \frac{d(2e^{-2t}\cosh 3t)}{dt} + 2(2e^{-2t}\cosh 3t)$$

$$3x = \cancel{-4e^{-2t}\cosh 3t} + 2e^{-2t}\,3\sinh 3t$$
$$\cancel{+4e^{-2t}\cosh 3t}$$

$$3x = 6e^{-2t}\sinh 3t$$

$$x = \frac{6e^{-2t}\sinh 3t}{3}$$

$$\boxed{x = 2e^{-2t}\sinh 3t}$$

therefore the equation:

$$\boxed{x = 2e^{-2t}\sinh 3t}$$
$$\boxed{y = 2e^{-2t}\cosh 3t}$$

UNIT - 4

2 marks

① find scalar Point function?

a function $\phi(x, y, z)$ is called a Scalar Point function. If at every Point (x, y, z) in space It assosiates a scalar and no derection.

② If $\vec{F} = t^3 \vec{i} + t^2 \vec{j} + (3t+1)\vec{k}$ find $\frac{d^2 \vec{F}}{dt^2}$

Given that

$$\vec{F} = t^3 \vec{i} + t^2 \vec{j} + (3t+1)\vec{k}$$

To find $\frac{d^2 \vec{F}}{dt^2}$

$$\frac{d(\vec{F})}{dt} = 3t^2 \vec{i} + 2t \vec{j} + 3\vec{k}$$

$$\boxed{\frac{d^2 \vec{F}}{dt^2} = 6t \vec{i} + 2\vec{j}}$$

③ grad ϕ where $\phi = x^2 y^3 z^4$ at $(1, 1, 1)$

$$\phi = x^2 y^3 z^4$$

$$\text{grad } \phi = \nabla \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$

$$= \vec{i}(2x y^3 z^4) + \vec{j}(3y^2 x^2 z^4) + \vec{k}(4z^3 x^2 y^3) \text{ at } (1,1,1)$$

$$\boxed{\nabla \phi = 2\vec{i} + 3\vec{j} + 4\vec{k}}$$

(4) To find curl ?

for a given vector function $\vec{F}$ curl of $\vec{F}$ is diffind as $\boxed{\nabla \times \vec{F}}$ and it is denoted by $\boxed{\text{curl } \vec{F}}$

$$\nabla \times \vec{F} = \text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix}$$

(5) find the solenoidel ?

If $\nabla \cdot \vec{F} = 0$ then $\vec{F}$ is a be solenoidel.

$$\nabla \cdot \vec{F} = \vec{i}\,\frac{\partial \vec{F_1}}{\partial x} + \vec{j}\,\frac{\partial \vec{F_2}}{\partial y} + \vec{k}\,\frac{\partial \vec{F_3}}{\partial z} = 0$$

(6) If $\vec{A}$ and $\vec{B}$ are irrotational show that the $\vec{A} \times \vec{B}$ is solenoidel.

given that if $\vec{A}$ and $\vec{B}$ irrotational then curl $\vec{A} = 0$, curl $\vec{B} = 0$,

check IF $\vec{A} \times \vec{B}$ if solenoidel

$$\text{div}(\vec{A} \times \vec{B}) = \vec{B} \cdot \text{curl } \vec{A} - \vec{A} \cdot \text{curl } \vec{B}$$

$$= \vec{B} \cdot 0 - \vec{A} \cdot 0$$

$$= 0 \cdot 0$$

$$\boxed{\text{div}(\vec{A} \times \vec{B}) = 0}$$

therefore $\vec{A} \times \vec{B}$ is solinoidel.

5 marks

① Prove that $\nabla r^n = n r^{n-2} \vec{r}$

Given that

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\nabla r^n = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \cdot r^n$$

$$= \vec{i}\, n r^{n-1} \frac{\partial r}{\partial x} + \vec{j}\, n r^{n-1} \frac{\partial r}{\partial y} + \vec{k}\, n r^{n-1} \frac{\partial r}{\partial z}$$

$$= n r^{n-1} \left(\vec{i}\frac{\partial r}{\partial x} + \vec{j}\frac{\partial r}{\partial y} + \vec{k}\frac{\partial r}{\partial z}\right)$$

$$= n r^{n-1} \left(\frac{x\vec{i}}{r} + \frac{y\vec{j}}{r} + \frac{z\vec{k}}{r}\right)$$

$$= n r^{n-1} \left(\frac{x\vec{i} + y\vec{j} + z\vec{k}}{r}\right)$$

$$= n r^{n-1} \left(\frac{\vec{r}}{r}\right)$$

$$= n r^{n-2} \vec{r}$$

$$\therefore \boxed{\nabla r^n = n r^{n-2} \vec{r}}$$

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(2) show that the surface $5x^2+2y-9x$ and $4x^2y+z^3-4=0$ are orthogonal at $(1,-1,2)$

the angle between the surface is the same as the angle between their normal

Normal to 1 surface $\nabla\phi_1 = \left(\vec{i}\frac{\partial\phi_1}{\partial x} + \vec{j}\frac{\partial\phi_1}{\partial y} + \vec{k}\frac{\partial\phi_1}{\partial z}\right)$

$= (10x-9)\vec{i} + 2\vec{j}$

$\hat{n}_1 = \nabla\phi(1,-1,2)$

$= \vec{i} + 2\vec{j}$

Normal to second surface $\nabla\phi_2 = \left(\vec{i}\frac{\partial\phi_2}{\partial x} + \vec{j}\frac{\partial\phi_2}{\partial y} + \vec{k}\frac{\partial\phi_2}{\partial z}\right)$

$= (8xy)\vec{i} + 4x^2\vec{j} + 3z^2\vec{k}$

$\hat{n}_2 = \nabla\phi(1,-1,2)$

$= -8\vec{i} + 4\vec{j} + 12\vec{k}$

$\vec{n}_1 \cdot \vec{n}_2 = (\vec{i}+2\vec{j}) \cdot (-8\vec{i} + 4\vec{j} + 12\vec{k})$

$= -8+8$

$= 0$

therefore, the surfaces are orthogonal.

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(3) Find the directional derivative of the function $x^2+y^2+z^2$ at $(2,2,1)$ in the direction $2\vec{i}+2\vec{j}+\vec{k}$

Given $\phi = x^2+y^2+z^2$ and the direction vector is $\vec{a} = 2\vec{i}+2\vec{j}+\vec{k}$. the directional derivative is given by $\nabla\phi . \vec{a}$

$$\nabla\phi = \left(\vec{i}\frac{\partial}{\partial x}+\vec{j}\frac{\partial}{\partial y}+\vec{k}\frac{\partial}{\partial z}\right)(x^2+y^2+z^2) = 2x\vec{i}+2y\vec{j}+2z\vec{k}$$

$$\nabla\phi(2,2,1) = 4\vec{i}+4\vec{j}+2\vec{k}$$

$$\frac{\nabla\phi . \vec{a}}{|\vec{a}|} = \frac{(4\vec{i}+4\vec{j}+2\vec{k}).(2\vec{i}+2\vec{j}+\vec{k})}{\sqrt{4+4+1}}$$

$$= \frac{8+8+2}{\sqrt{9}}$$

$$= 18/3$$

$$\boxed{\frac{\nabla\phi . \vec{a}}{|\vec{a}|} = 3}$$

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4) Find the equation to tangent plane to the surface

$z = x^2 + y^2 \quad (2, -1, 5)$

First let as find the normal $\nabla\phi$:

$$\nabla\phi = \vec{i}\,\frac{\partial\phi}{\partial x} + \vec{j}\,\frac{\partial\phi}{\partial y} + \vec{k}\,\frac{\partial\phi}{\partial z}$$

$$\nabla\phi = 2x\vec{i} + 2y\vec{j} - \vec{k}$$

$$\nabla\phi\,(2,-1,5) = 4\vec{i} - 2\vec{j} - \vec{k}$$

to the tangent plane

$$(\vec{r} - \vec{a}) \cdot \nabla\phi = 0$$

$$(x\vec{i} + y\vec{j} + z\vec{k}) - (2\vec{i} - \vec{j} + 5\vec{k}) \cdot 4\vec{i} - 2\vec{j} - \vec{k} = 0$$

$$\vec{i}\,(x-2) + \vec{j}\,(y+1) + \vec{k}\,(z-5) \cdot 4\vec{i} - 2\vec{j} - \vec{k} = 0$$

$$4x - 8 - 2y - 2 - z + 5 = 0$$

$$\boxed{4x - 2y - z = 5}$$

therefore the tangent Plane:

$$\boxed{4x - 2y - z = 5}$$

5) Find ϕ if $\nabla\phi = 8xy\vec{i} + 4x^2\vec{j} + 3z^2\vec{k}$

Given that

$$\nabla\phi = 8xy\vec{i} + 4x^2\vec{j} + 3z^2\vec{k}$$

$$\left(\vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}\right) = 8xy\vec{i} + 4x^2\vec{j} + 3z^2\vec{k}$$

$$\frac{\partial\phi}{\partial x} = 8xy, \quad \frac{\partial\phi}{\partial y} = 4x^2, \quad \frac{\partial\phi}{\partial z} = 3z^2$$

Integrating with respect to x, y, z repectively.

$\phi = 4x^2y + f(y,z)$ since y, z are treated as constants

$\phi = 4x^2y + f(x,z)$ since x, z are treated as constants

$\phi = z^3 + f(x,y)$ since x, y are treated as constants

$\therefore \phi = 4x^2y + z^3 + c$ where c is a constant

6) Evaluate $\nabla \log r$ where $r = |\vec{r}|$; $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$r = |\vec{r}| = \sqrt{x^2+y^2+z^2}$$

$$r^2 = x^2+y^2+z^2$$

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$$2r \frac{\partial r}{\partial x} = 2x$$

$$\frac{\partial r}{\partial x} = x/r$$

$|||^{ly}$

$$\frac{\partial r}{\partial y} = y/r$$

$$\frac{\partial r}{\partial z} = z/r$$

To find $\nabla \log r$: $\nabla \log r = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)(\log r)$

$$\nabla \log r = \vec{i}\ 1/r \cdot \frac{\partial r}{\partial x} + \vec{j}\ 1/r \frac{\partial r}{\partial y} + \vec{k}\ 1/r \frac{\partial r}{\partial z}$$

$$= \vec{i}\frac{1}{r} x/r + \vec{j}\ 1/r\ y/r + \vec{k}\frac{1}{r}\frac{z}{r}$$

$$= \frac{x\vec{i}}{r^2} + \frac{y\vec{j}}{r^2} + \frac{z\vec{k}}{r^2}$$

$$= \frac{x\vec{i} + y\vec{j} + z\vec{k}}{r^2}$$

$$= r/r^2$$

$$\therefore \boxed{\nabla \log r = r/r^2}$$

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7) Find div $\vec{F}$ and curl $\vec{F}$ at $(1, 2, -3)$ where

$$\vec{F} = (x^2 - y^2 + 2xz)\vec{i} + (xz - xy + yz)\vec{j} + (z^2 + x^2)\vec{k}$$

$$\vec{F} = (x^2 - y^2 + 2xz)\vec{i} + (xz - xy + yz)\vec{j} + (z^2 + x^2)\vec{k}$$

find div $\vec{F}$:

$$\nabla \vec{F} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \cdot \Big((x^2 - y^2 + 2xz)\vec{i}$$

$$+ (xz - xy + yz)\vec{j} + (z^2 + x^2)\vec{k} \Big)$$

$$\nabla \vec{F} = \frac{\partial x^2 - y^2 + 2xz}{\partial x} + \frac{\partial xz - xy + yz}{\partial y} + \frac{\partial z^2 + x^2}{\partial z}$$

$$= (2x + 2z) + (-x + z) + (2z)$$

at $(1, 2, -3)$

$$\nabla \cdot \vec{F} = -4 - 4 - 6$$

$$\boxed{\nabla \cdot \vec{F} = -14}$$

Find curl $\vec{F}$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 - y^2 + 2xz & xz - xy + yz & z^2 + x^2 \end{vmatrix}$$

$$= \vec{i}\left(0-(x+y)\right) - \vec{j}\left(2x-2x\right) + \vec{k}\left((z-y)+(2y)\right)$$

at $(1, 2, -3)$

$$= \vec{i}\left(-1-2\right) - \vec{j}\left(0\right) + \vec{k}\left(-3-2+4\right)$$

$$\boxed{\nabla \times \vec{F} = -3\vec{i} - \vec{k}}$$

10 marks

① Prove $\operatorname{div}(r^n \vec{r}) = (n+3)r^n$ and $\operatorname{div}\left(\frac{\vec{r}}{r}\right) = 2/r$

$\operatorname{div}(r^n \vec{r}) = (n+3)r^n$

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$r = |\vec{r}| = \sqrt{x^2+y^2+z^2}$$

$$r^2 = x^2+y^2+z^2$$

$$2r \frac{\partial r}{\partial x} = 2x$$

$$\frac{\partial r}{\partial x} = x/r$$

similarly

$$\frac{\partial r}{\partial y} = y/r \ , \ \frac{\partial r}{\partial z} = z/r$$

$$r^n \vec{r} = r^n x\vec{i} + r^n y\vec{j} + r^n z\vec{k}$$

$$\operatorname{div}(r^n \vec{r}) = \left[\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right] \cdot \left[r^n x\vec{i} + r^n y\vec{j} + r^n z\vec{k}\right]$$

$$= \left[\frac{\partial r^n x}{\partial x} + \frac{\partial r^n y}{\partial y} + \frac{\partial r^n z}{\partial z}\right]$$

$$= \left[n r^{n-1}\frac{\partial r}{\partial x} x + r^n\right] + \left[n r^{n-1}\frac{\partial r}{\partial y} y + r^n\right] + \left[n r^{n-1}\frac{\partial r}{\partial z} z + r^n\right]$$

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$$= \left[n r^{n-1} \left(\frac{x}{r}\right) x + r^n \right] + \left[n r^{n-1} \left(\frac{y}{r}\right) y + r^n \right] + \left[n r^{n-1} \left(\frac{z}{r}\right) z + r^n \right]$$

$$= n r^{n-1} \frac{x^2+y^2+z^2}{r} + 3r^n = n r^{n-1} \frac{r^2}{r} + 3r^n$$

$$= n r^n + 3r^n$$

$$\therefore \boxed{\operatorname{div}(r^n \vec{r}) = (n+3) r^n}$$

$$\underline{\operatorname{div}\left(\frac{\vec{r}}{r}\right) = 2/r}$$

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\nabla \cdot \left(\frac{\vec{r}}{r}\right) = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \cdot \left(\frac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= \cancel{\frac{\partial}{\partial x}\left(x\vec{i} + y\vec{j}\right.} = \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= \frac{\sqrt{x^2+y^2+z^2} - \frac{x^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2} + \frac{\sqrt{x^2+y^2+z^2} - \frac{y^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}$$

$$+ \frac{\sqrt{x^2+y^2+z^2} - \frac{z^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}$$

$$= \frac{\frac{x^2+y^2+z^2-x^2+x^2+y^2+z^2-y^2+x^2+y^2+z^2-z^2}{\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}$$

$$= \frac{\frac{2(x^2+y^2+z^2)}{r}}{r^2}$$

$$= \frac{2\cancel{r^2}}{r} \times \frac{1}{\cancel{r^2}}$$

$$= 2/r$$

$$\therefore \boxed{div\left(\frac{\vec{r}}{r}\right) = 2/r}$$

2) Prove the $\vec{F} = (2x+yz)\vec{i} + (4y+zx)\vec{j} - (6z-xy)\vec{k}$ is irrotational. Find the scalar potential of. $\vec{F}$

$$\vec{F} = (2x+yz)\vec{i} + (4y+zx)\vec{j} - (6z-xy)\vec{k}.$$

$$\text{curl } \vec{F} = \nabla \times \vec{F} = 0$$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (2x+yz) & (4y+zx) & (-6z+xy) \end{vmatrix}$$

$$= \vec{i}\left[\frac{\partial}{\partial y}(-6z+xy) - \frac{\partial}{\partial z}(4y+zx)\right]$$

$$- \vec{j}\left[\frac{\partial}{\partial x}(-6z+xy) - \frac{\partial}{\partial z}(2x+yz)\right]$$

$$+ \vec{k}\left[\frac{\partial}{\partial x}(4y+zx) - \frac{\partial}{\partial y}(2x+yz)\right]$$

$$= \vec{i}[x-x] - \vec{j}[y-y] - \vec{k}[z-z]$$

$$\text{Curl } \vec{F} = 0$$

therefore the equation is irrotational.

If $\vec{F} = \nabla\phi$ then ϕ is called the scalar potential of $\vec{F}$

$$\left(\vec{i}\frac{\partial\phi}{\partial x} + \vec{j}\frac{\partial\phi}{\partial y} + \vec{k}\frac{\partial\phi}{\partial z}\right) = (2x+yz)\vec{i} + (4y+zx)\vec{j} - (6z-xy)\vec{k}$$

Integrating with respect to x, y, z respectively

$$\phi = x^2 + xyz + f(y,z)$$

$$\phi = 2y^2 + xyz + f(x,z)$$

$$\phi = -3z^2 + xyz + f(x,y)$$

$$\boxed{\phi = x^2 + 2y^2 - 3z^2 + xyz + C}$$

3) Determine the constant a so that the vector $\vec{F} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ is solenoidal.

A vector $\vec{F}$ is solenoidal if $\operatorname{div} \vec{F} = 0$

$$\operatorname{div} \vec{F} = \nabla \cdot \vec{F} = 0$$

$$\nabla \cdot \vec{F} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \cdot \left((x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}\right)$$

$$\Rightarrow \frac{\partial(x+3y)}{\partial x} + \frac{\partial(y-2z)}{\partial y} + \frac{\partial(x+az)}{\partial z} = 0$$

$$\Rightarrow 1 + 1 + a = 0$$

$$\Rightarrow 2 + a$$

$$\boxed{a = -2}$$

$$\Rightarrow \frac{\partial(x+3y)}{\partial x} + \frac{\partial(y-2z)}{\partial y} + \frac{\partial(x \overset{-2}{\cancel{+a}} z)}{\partial z}$$

$$= 1 + 1 - 2$$

$$\nabla \cdot \vec{F} = 0$$

therefore $\vec{F}$ is solenoidal.

2. Mark.

1. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = y\vec{i} - x\vec{j}$ and C is the curve $x = y^2$ from $(0,0)$ to $(4,2)$.

$$\int_C \vec{F} \cdot d\vec{r} = \int_C (y\vec{i} - x\vec{j}) \cdot (dx\vec{i} + dy\vec{j})$$

$$= \int_C (y\,dx - x\,dy) = \int_0^4 \sqrt{x}\,dx - \int_0^2 y^2\,dy$$

$$= \int_0^4 x^{\frac{1}{2}}\,dx - \int_0^2 y^2\,dy = \left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 - \left.\frac{y^3}{3}\right]_0^2$$

$$= \frac{2}{3} 4^{\frac{3}{2}} - \frac{2^3}{3} = \frac{16}{3} - \frac{8}{3}$$

$$= \frac{8}{3}.$$

2. Green's Theorem:

If $P(x,y)$ and $Q(x,y)$ are function on the regio bounded by simple closed curve in the x-y plane where P, Q have continuous partial derivatives then

$$\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy.$$

3. Gauss's Divergence Theorem:

The surface integral of the normal component of a vector function $\vec{F}$ taken over a closed surface S is equal to the integral of the divergence of $\vec{F}$ taken over the volume V enclosed by the surface S.

$$\iint_S \vec{F} \cdot \hat{n}\, ds = \iiint_V (\nabla \cdot \vec{F})\, dV.$$

4. Stoke's Theorem:

The surface integral of the component of curl $\vec{F}$ along the normal to the surface S taken over the surface S bounded by the curve c is equal to the line integral of the vector function $\vec{F}$ taken along the closed curve c.

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, ds.$$

5. Volume Integral:

If $\vec{F}$ is a function defined on a surface enclosing a volume V, volume integral is given by $\iiint_V \vec{F} \cdot d\vec{v}$.

5 Mark.

1. Using Green's theorem evaluate $\int_C x^2 y\,dx + x^2\,dy$ where C is the triangle joining the points (0,0) (1,0) and (0,1).

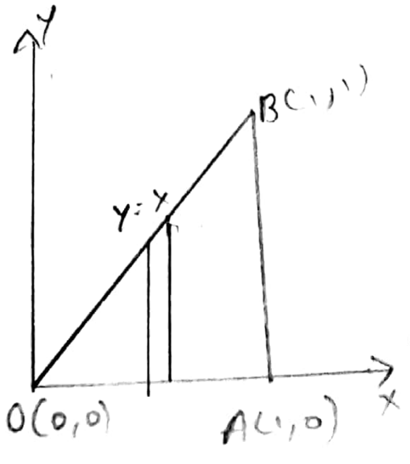
By Green's theorem.

$$\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,.dy$$

$$P = x^2 y\,;\ Q = x^2.$$

$$\int x^2 y\,dx + x^2\,dy = \iint_R (2x - x^2)\,dx\,dy$$

$$= \int_0^1 \int_0^x (2x - x^2)\,dy\,dx$$



$$= \int_0^1 (2x - x^2)(y)_0^x\,dx = \int_0^1 (2x^2 - x^3)\,dx$$

$$= \left[2\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}.$$

2. Verify Green's theorem in the plane for $\int_C (3xy + 2x)\,dx + 2y^2\,dy$ where C is the curve bounded by $x = 0$; $y = 0$ and $x + y = 1$.

By Green's theorem.

$$\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$P = 3xy + 2x \, ; \quad Q = 2y^2$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 - 3x = -3x$$

$$\text{R.H.S} = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$= \iint_R (-3x)\,dx\,dy = -3\int_0^1 \int_0^{1-x} (x)\,dy\,dx$$

$$= -3\int_0^1 x(1-x)\,dx = -3\int_0^1 (x - x^2)\,dx$$

$$= -\frac{1}{2}$$

$$\text{L.H.S} = \int_C P\,dx + Q\,dy$$

$$= \int_{OA} 3xy\,dx + 2y^2\,dy + \int_{AB} 3xy\,dx + 2y^2\,dy + \int_{BO} 3xy\,dx + 2y^2\,dy$$

Along OA, $y = 0$; and along AB, $y = 1 - x$ and along BO, $x = 0$

$$= 0 + \int_{AB} 3x(1-x)\,dx + 2y^2\,dy + \int_{BO} 2y^2\,dy$$

$$= \int_1^0 (3x - 3x^2)\,dx + \int_0^1 2y^2\,dy + \int_1^0 2y^2\,dy$$

$$= -\int_0^1 (3x - 3x^2)\,dx + \int_0^1 2y^2\,dy - \int_0^1 2y^2\,dy$$

$$= -3\left[\frac{x^2}{2} - \frac{x^3}{3}\right] + 0 = -3\left[\frac{1}{2} - \frac{1}{3}\right] = -\frac{1}{2}$$

Hence Green's theorem verified.

3. Evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$ where $\vec{F} = xy\vec{i} - x^2\vec{j} + (x+z)\vec{k}$ and S is the plane $2x + 2y + z = 6$ in the first octant.

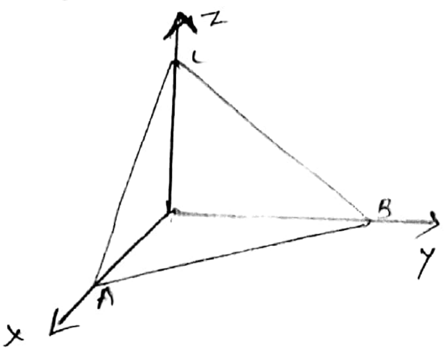
First find the unit normal to the plane $2x + 2y + z = 6$

$$\hat{n} = \frac{2\vec{i} + 2\vec{j} + \vec{k}}{3}$$

$$\vec{F}\cdot\hat{n} = \frac{1}{3}\left[2xy - 2x^2 + (x+z)\right]$$

$$= \frac{1}{3}\left[2xy - 2x^2 + x + 6 - 2x - 2y\right]$$

$$= \frac{1}{3}\left[2xy - 2x^2 - x - 2y + 6\right]$$



Taking projection of the surface on the x-y plane

then $ds = \dfrac{dx\,dy}{\hat{n}\cdot\vec{k}}$

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \iint_R \frac{1}{3}\left[2xy - 2x^2 - x - 2y + 6\right]\frac{dx\,dy}{\frac{1}{3}}$$

$$= \int_0^3\int_0^{3-x} (2xy - 2x^2 - x - 2y + 6)\,dy\,dx$$

$$= \int_0^3 \left[2x\left[\frac{y^2}{2}\right] - 2x^2[y] - x[y] - 2\left[\frac{y^2}{2}\right] + 6[y]\right]_0^{3-x} dx$$

$$= \int_0^3 x(3-x)^2 - 2x^2(3-x) - x(3-x) - (3-x)^2 + 6(3-x)\,dx$$

$$= \int_0^3 (3-x)\left[x(3-x) - 2x^2 - x - (3-x) + 6\right]dx$$

$$= \int_0^3 (3-x)\left[3x - 3x^2 + 3\right]dx$$

$$= 3\int_0^3 (2x - 4x^2 + x^3 + 3)\,dx$$

$$= 3\left[x^2 - \frac{4}{3}x^3 + \frac{x^4}{4} + 3x\right]_0^3 = 3\left[\frac{9}{4}\right]$$

$$\iint \vec{F}\cdot\hat{n}\,ds = \frac{27}{4}.$$

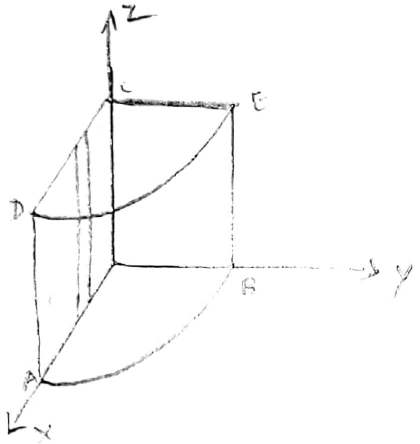
④ Evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$ where $\vec{F} = yz\vec{i} + 2y^2\vec{j} + xz^2\vec{k}$ and S is the surface in the cylinder $x^2+y^2=9$ in the First octant $z=0$ and $z=2$.

Sol:- First the unit normal to the cylinder $x^2+y^2-9=0$.

$$\hat{n} = \frac{2x\vec{i}+2y\vec{j}}{\sqrt{4(x^2+y^2)}}$$

$$= \frac{2x\vec{i}+2y\vec{j}}{\sqrt{4(9)}} = \frac{x\vec{i}+y\vec{j}}{3}$$

$$\vec{F}\cdot\hat{n} = \frac{1}{3}\left[xyz+2y^3\right]$$



Taking projection of the surface on the x-y plane then $ds = \frac{dx\,dz}{\hat{n}\cdot\hat{j}}$

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \iint_R \frac{1}{3}\left[xyz+2y^3\right]\frac{dx\,dz}{\left(\frac{y}{3}\right)}$$

$$= \iint_R \left[xz+2y^2\right]dx\,dz = \int_0^2\int_0^3 xz+2(9-x^2)\,dx\,dz$$

$$= \int_0^2 \left[z\left[\frac{x^2}{2}\right]+18\left[x\right]-2\left[\frac{x^3}{3}\right]\right]_0^3 dz$$

$$= \int_0^2 \left[\frac{9}{2} z + 54 - 18 \right] dz = \frac{9}{2} \left[\frac{z^2}{2} \right]_0^2 + 36 [z]_0^2$$

$$= \frac{9}{2} [2] + 36 [2] = 81$$

10 Mark

1. verify Green's theoram $\int_c (x^2-y^2)\,dx + 2xy\,dy$ where c is the boundary of the region bounded by the lines $x=0,\ x=a,\ y=0,\ y=b$.

By Green's theoram $\int_c p\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$.

$$P = x^2 - y^2;\ Q = 2xy$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2y - (-2y)$$

$$= 4y$$

The region is rectangle in the x-y plane bounded by $x=0,\ y=0,\ x=a,\ y=a$.

$$R.H.S = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

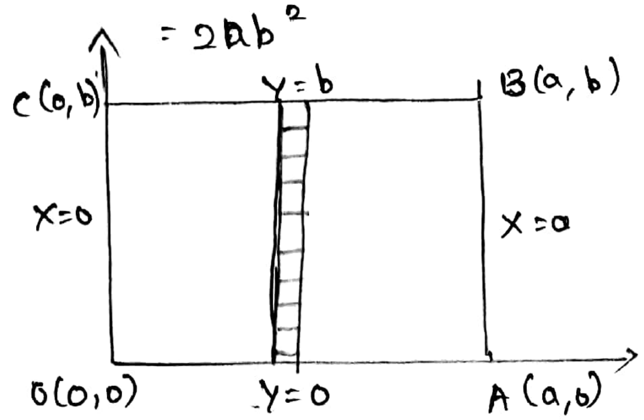
$$= \iint_R (4y)\,dx\,dy$$

$$= 4\int_0^a \int_0^b (y)\,dy\,dx$$

$$= 4\int_0^a \left[\frac{y^2}{2}\right]_0^b dx$$

$$= 2b^2 [x]_0^a$$

$$= 2ab^2$$



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$$L.H.S = \int_C P\,dx + Q\,dy$$

$$= \int_{OA} (x^2-y^2)dx + 2xy\,dy + \int_{AB} (x^2-y^2)dx + 2xy\,dy$$

$$+ \int_{BC} (x^2-y^2)dx + 2xy\,dy + \int_{CO} (x^2-y^2)dx + 2xy\,dy$$

Along OA, $y=0$, $dy=0$ and along AB, $x=a$, $dx=0$ and along BC, $y=b$, $dy=0$ and along CO $x=0$; $dx=0$

$$= \int_{OA} (x^2)dx + \int_{AB} 2ay\,dy + \int_{BC} (x^2-b^2)dx + 0$$

$$= \int_0^a x^2 dx + 2a\int_0^b y\,dy + \int_a^0 x^2 dx - \int_a^0 b^2 dx$$

$$= 2a\left[\frac{y^2}{2}\right]_0^b + b^2[x]_0^a = ab^2 + ab^2 = 2ab^2$$

Hence Green's theoram verified

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2. verify Gauss's divergence theoram given that $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ and S is the surface of the cube $x=0$, $x=1$, $y=0$, $y=1$, $z=0$ and $z=1$

By Gauss's theoram $\iint_S \vec{F}.\hat{n}ds = \iiint_V (\nabla.\vec{F})dv$

$$R.H.S = \iiint_V (\nabla.\vec{F})dv$$

$$= \iiint \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial x} + \vec{k}\frac{\partial}{\partial z}\right).\left(4xz\vec{i} - y^2\vec{j} + yz\vec{k}\right)dv$$

$$= \iiint (4z - 2y + y)dx\, dy\, dz$$

$$= \int_0^1\int_0^1\int_0^1 (4z - y)dx\, dy\, dz$$

$$= \int_0^1\int_0^1 (4z - y)(x)_0^1 dy\, dz$$

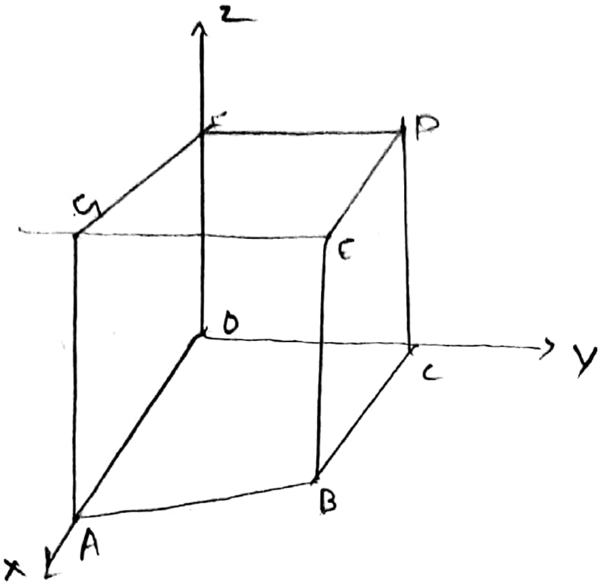
$$= \int_0^1 4z(y)_0^1 - \left(\frac{y^2}{2}\right]_0^1 dz$$

$$= \int_0^1 \left(4z - \frac{1}{2}\right)dx$$

$$= 4\left[\frac{z^2}{2}\right]_0^1 - \frac{1}{2}(z)_0^1$$

$$= 2 - \frac{1}{2} = \frac{3}{2}$$

Now let us evaluate the L.H.S



$$L.H.S = \iint_S \vec{F}\cdot\hat{n}\,ds$$

Over the surface OABC, $z=0$

$dz=0$ and $\hat{n}=-\hat{k}$ since the unit normal is taken outward $ds=dx\,dy$.

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \int_0^1\int_0^1 -y^2\hat{j}\cdot(-\hat{k})\,dx\,dy$$

$$=0$$

Over the surface DEGF, $z=1$

$dz=0$ and $\hat{n}=\vec{k}$ since the unit normal is taken outward $ds=dx\,dy$

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \int_0^1\int_0^1 4x\vec{i}-y^2\vec{j}+yk\vec{k}\cdot(\vec{k})\,dx\,dy$$

$$=\int_0^1\int_0^1 y\,dx\,dy$$

$$=(x)\left[\frac{y^2}{2}\right]_0^1=\frac{1}{2}$$

Over the surface BCDE, $y=1$

$dy=0$ and $\hat{n}=j$

$ds=dx\,dz$

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \int_0^1\int_0^1 4xzi - \hat{j} + zk\cdot(j)\,dx\,dz$$

$$= \int_0^1\int_0^1 -\,dx\,dz$$

$$= -1$$

Over the surface AOGF, $y=0$

$dy=0$ and $\hat{n}=-j$

$ds = dx\,dz$

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \int_0^1\int_0^1 4xzi\cdot(j)\,dx\,dz$$

$= 0$ since $i\cdot j = 0$

Over the surface OCDG, $x=0$

$dx=0$ and $\hat{n}=-i$

$ds = dy\,dz$

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \int_0^1\int_0^1 0i - y^2 j + yzk\cdot(-i)\,dy\,dz$$

$= 0$

Over the surface ABEF, $x=1$

$dx=0$ and $\hat{n}=1$, since the unit normal is taken outward $ds = dy\,dz$

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \int_0^1\int_0^1 4zi - y^2\hat{j} + yzk\cdot(i)\,dy\,dz$$

$$= \int_0^1\int_0^1 4z\,dz\,dy$$

$$= (y)\left[\frac{4z^2}{2}\right]_0^1$$

$$= 2$$

$$\therefore \iint_S \vec{F}\cdot\hat{n}\,ds = 0+\frac{1}{2}-1+0+0+2$$

$$= \frac{3}{2}$$

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \frac{3}{2} = \iiint_V (\nabla\cdot\vec{F})\,dv$$

Therefore, Gauss's theoram Verified.

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3. Verify stoke's theoram for $\vec{F} = (x^2-y^2)\vec{i} + 2xy\vec{j}$ where s is the region bounded by $x=0$, $x=a$, $y=b$, $y=0$.

By stroke's theoram $\int_c \vec{F}\cdot d\vec{r} = \iint_s (\nabla\times\vec{F})\cdot\hat{n}\,ds$

$$R.H.S = \iint_s (\nabla\times\vec{F})\cdot\hat{n}\,ds$$

The unit normal to the surface which is rectangle in the $x-y$ plane is $\hat{n} = \vec{k}$

$$\nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2-y^2 & 2xy & 0 \end{vmatrix}$$

$$= \vec{i}[0-0] - \vec{j}[0-0] + \vec{k}[4y]$$

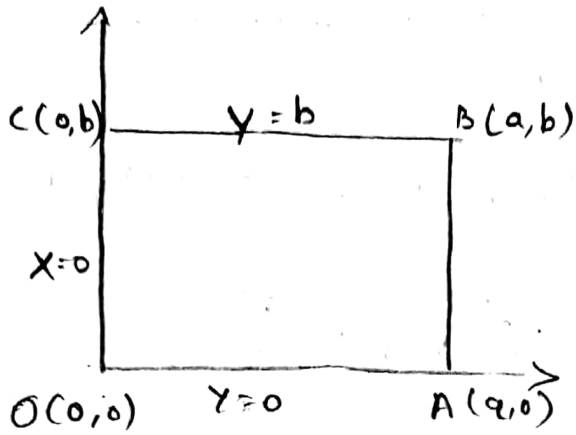
$$= 4y\vec{k}$$

$$R.H.S = \iint_s (\nabla\times\vec{F})\cdot\hat{n}\,ds$$

$$= \iint 4y\hat{k}\cdot\hat{n}\,ds = \iint 4y\vec{k}\cdot(\hat{k})\frac{dx\,dy}{\hat{n}\cdot\vec{k}}$$

$$= \int_0^a\int_0^b 4y\,dy\,dx = \int_0^a 4\left[\frac{y^2}{2}\right]_0^b dx$$

$$= \int_0^a 2b^2\,dx = 2ab^2$$



$$L.H.S = \int_C \vec{F}\cdot\vec{dr}$$

$$= \int_{OA} \vec{F}\cdot\vec{dr} + \int_{AB} \vec{F}\cdot\vec{dr} + \int_{BC} \vec{F}\cdot\vec{dr} + \int_{CO} \vec{F}\cdot\vec{dr}$$

$$\int_C \vec{F}\cdot\vec{dr} = \int_C \left((x^2-y^2)\,i + 2xy\,j\right)\cdot(dx\,i + dy\,j + dz\,k)$$

$$= \int_C (x^2-y^2)\,dx + 2xy\,dy$$

$$\int_{OA} \vec{F}\cdot\vec{dr} = \int_{OA} (x^2-y^2)\,dx + 2xy\,dy$$

$$= \int_0^a x^2\,dx \quad [\text{since along } OA,\ y=0]$$

$$= \frac{a^3}{3}$$

$$\int_{AB} \vec{F}\cdot\vec{dr} = \int_{AB} (x^2-y^2)\,dx + 2xy\,dy$$

$$= \int_0^b 2ay\,dy \quad [\text{since along } AB,\ x=a;\ dx=0]$$

$$= ab^2$$

$$\int_{BC} \vec{F}\cdot\vec{dr} = \int_{BC} (x^2-y^2)\,dx + 2xy\,dy$$

$$= \int_a^0 (x^2-b^2)\,dx \quad [\text{since along } BC,\ y=b;\ dy=0]$$

$$= \left[\left(\frac{x^3}{3}\right) - b^2(x)\right]_a^0$$

$$= \left[\left(0-\frac{a^3}{3}\right) - b^2(0-a)\right]$$

$$= -\frac{a^3}{3} + ab^2$$

$$\int_{CO} \vec{F}\cdot\vec{dr} = \int_{CO} (x^2-y^2)\,dx + 2xy\,dy$$

$$= \int_a^0 0 + 0\,dy \quad [\text{since along } CO,\ x=0;\ dx=0]$$

$$= 0$$

$$L.H.S = \int_C \vec{F}\cdot\vec{dr}$$

$$= \int_{OA} \vec{F}\cdot\vec{dr} + \int_{AB} \vec{F}\cdot\vec{dr} + \int_{BC} \vec{F}\cdot\vec{dr} + \int_{CO} \vec{F}\cdot\vec{dr}$$

$$= \frac{a^3}{3} + ab^2 - \frac{a^3}{3} + ab^2 + 0$$

$$= 2ab^2$$

$$\therefore \int_C \vec{F}\cdot\vec{dr} = 2ab^2 = \iint_S (\nabla\times\vec{F})\cdot\hat{n}\,ds$$

4. verify stokes theoram for $\vec{F} = (2x-y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper half of the sphere $x^2+y^2+z^2=1$

By stoke's theoram $\int_C \vec{F}\cdot d\vec{r} = \iint_S (\nabla\times\vec{F})\cdot\hat{n}\,ds$

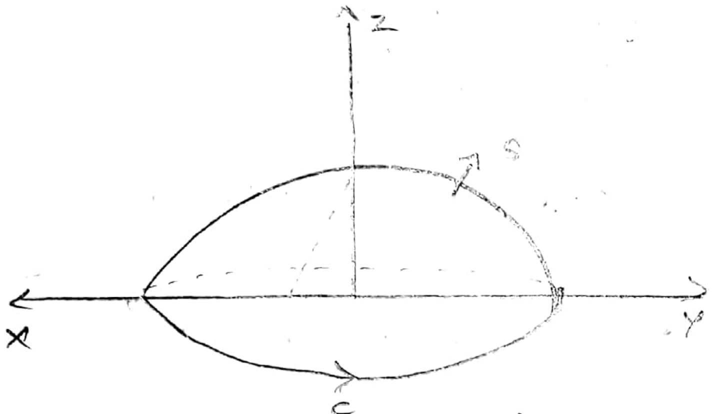
$$R.H.S = \iint_S (\nabla\times\vec{F})\cdot\hat{n}\,ds$$

The unit normal to the surface $x^2+y^2+z^2=1$.

$$\hat{n} = \frac{2x\vec{i}+2y\vec{j}+2z\vec{k}}{\sqrt{4(x^2+y^2+z^2)}}$$

$$= \frac{2x\vec{i}+2y\vec{j}+2z\vec{k}}{\sqrt{4}}$$

$$= x\vec{i}+y\vec{j}+z\vec{k}$$



$$\nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x-y & -yz^2 & -y^2z \end{vmatrix}$$

$$= \vec{i}[2yz-2yz] - \vec{j}[0-0] + \vec{k}[0+1]$$

$$= \vec{k}$$

$$R.H.S = \iint_S (\nabla\times\vec{F})\cdot\hat{n}\,ds$$

$$= \iint \vec{k}\cdot\hat{n}\,ds$$

Taking projection on x-y plane of the surface

$$= \iint \hat{k} \cdot (x\vec{i} + y\vec{j} + z\vec{k}) \frac{dx\,dy}{\hat{n} \cdot \vec{k}}$$

$$= \iint z \frac{dx\,dy}{z} = \iint dx\,dy$$

= Area in x-y plane

$$= \pi r^2 = \pi$$

$$L.H.S = \int_c \vec{F} \cdot d\vec{r} = \int_c \{(2x-y)\vec{i} - yz^2\vec{j} + y^2z\vec{k}\} \cdot (dx\vec{i} + dy\vec{j} + dz\vec{k})$$

$$= \int_c ((2x - y)dx),$$

Since on the x-y plane $z = 0$ and on the circular boundary

$x = \cos\theta$, $y = \sin\theta$.

$$= \int_c ((2\cos\theta - \sin\theta)(-\sin\theta\, d\theta))$$

$$= \int_0^{2\pi} \sin 2\theta\, d\theta + \int_0^{2\pi} \sin^2\theta\, d\theta$$

$$= 0 + 4\int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta$$

$$= 4\left(\frac{2-1}{2}\right)\left(\frac{\pi}{2}\right)$$

$$\therefore \int_c \vec{F} \cdot d\vec{r} = \pi = \iint_s (\nabla \times \vec{F}) \cdot \hat{n}\, ds$$

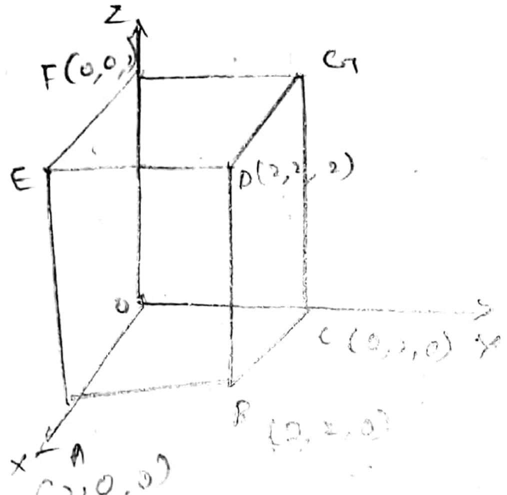
Hence Stoke's Theorem Verified.

5. Verify stoke's theoram for $\vec{F} = (y-z+2)\vec{i} + (yz+4)\vec{j} - xz\vec{k}$ where S is the open surface of the cube $x=0, x=2, y=0, y=2, z=2$ above the x-y plane

The given surface is bounded by five surfaces with boundary OABCO on the x-y plane

By stoke's theoram $\int_C \vec{F}.d\vec{r} = \iint_S (\nabla\times\vec{F}).\hat{n}\,ds$

$$R.H.S = \iint_S (\nabla\times\vec{F}).\hat{n}\,ds$$



$$\nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y-z+2 & yz+4 & -xz \end{vmatrix}$$

$$= \vec{i}[0-y] - \vec{j}[-z+1] + \vec{k}[0-1]$$

$$= -y\vec{i} + (z-1)\vec{j} - \vec{k}$$

on the surface ABDE, $x=2$, $\hat{n} = \vec{i}$

$$\iint_{ABDE} (\nabla\times\vec{F}).\hat{n}\,ds = \iint_{ABDE} -y\,\frac{dy\,dz}{\hat{n}.\vec{i}}$$

$$= -\int_0^2\int_0^2 y\,dy\,dz = -\int_0^2 \left[\frac{y^2}{2}\right]_0^2 dz$$

$$= \int_0^2 2\,dz = -2[z]_0^2 = -4$$

on the surface OCGF, $x=0$, $\hat{n} = -\vec{i}$

$$\iint_{OCGF} (\nabla\times\vec{F}).\hat{n}\,ds = \iint_{OCGF} -y\,\frac{dy\,dz}{\hat{n}.\vec{i}}$$

$$= \int_0^2\int_0^2 y\,dy\,dz = \int_0^2 \left[\frac{y^2}{2}\right]_0^2 dz$$

$$= \int_0^2 2\,dz = 2[z]_0^2 = 4$$

on the surface BCDG, $y=2$, $\hat{n} = \vec{j}$

$$\iint_{BCDG} (\nabla\times\vec{F}).\hat{n}\,ds = \iint_{BCDG} (z-1)\,\frac{dx\,dz}{\hat{n}.j}$$

$$= \int_0^2\int_0^2 (z-1)\,dx\,dz$$

$$= \int_0^2 \left[\frac{(z-1)^2}{2}\right]_0^2 dx$$

on the surface OAEF, $y=0$, $\hat{n} = -\vec{j}$

$$\iint_{OAEF} (\nabla\times\vec{F}).\hat{n}\,ds = -\iint_{OAEF} (z-1)\,\frac{dx\,dz}{\hat{n}.j}$$

$$= -\int_0^2\int_0^2 (z-1)\,dx\,dz$$

$$= -\int_0^2 \left[\frac{(z-1)^2}{2}\right]_0^2 dx$$

$$= 0$$

on the surface DGEF, $z=2$; $\hat{n}=\vec{k}$

$$\iint_{DGEF} (\nabla\times\vec{F}).\hat{n}\,ds = \iint_{DGEF} -1\,\frac{dx\,dy}{\vec{n}.k}$$

$$= -\int_0^2\int_0^2 dx\,dy$$

$$= -4$$

$$R.H.S = \iint_S (\nabla\times\vec{F}).\hat{n}\,ds$$

$$= -4+4+0+0-4$$

$$= -4$$

$$L.H.S = \int_C \vec{F}.\vec{dr}$$

$$= \int_{OA} \vec{F}.\vec{dr} + \int_{AB} \vec{F}.\vec{dr} + \int_{BC} \vec{F}.\vec{dr} + \int_{CO} \vec{F}.\vec{dr}$$

on OA, $y=0$, $z=0$, $dy=dz=0$,

$$\int_{OA} \vec{F}.\vec{dr} = \int_{OA} 2\,dx$$

$$= \int_0^2 2\,dx = 4$$

on AB, $x=2$, $z=0$, $dx=dz=0$,

$$\int_{AB} \vec{F}.\vec{dr} = \int_{AB} 4y\,dy$$

$$= 4\int_0^2 y\,dy = 4\left[\frac{y^2}{2}\right]_0^2 = 8$$

On BC, $y = 2$, $z = 0$, $dy = dz = 0$,

$$\int_{BC} \vec{F} \cdot d\vec{r} = \int_{AB} 4 dx$$

$$= 4 \int_{0}^{2} dx = -8$$

On CO, $x = 0$, $z = 0$, $dx = dz = 0$,

$$\int_{CO} \vec{F} \cdot d\vec{r} = \int_{CO} 4 dy$$

$$= 4 \int_{2}^{0} dy$$

$$= -8$$

$$L.H.S = \int_{c} \vec{F} \cdot d\vec{r}$$

$$= 4 + 8 - 8 - 8$$

$$= -4$$

$$\int_{c} \vec{F} \cdot d\vec{r} = -4 = \iint_{S} (\nabla \times \vec{F}) \cdot \hat{n} \, ds$$

Therefore, stoke's theoram verified.

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